Structural Mechanics (1)

Week No-04

Part-01

Deflection in Determinate Structures

B. Haidar

Deflections of Trusses, Beams, & Frames: Work-Energy Methods

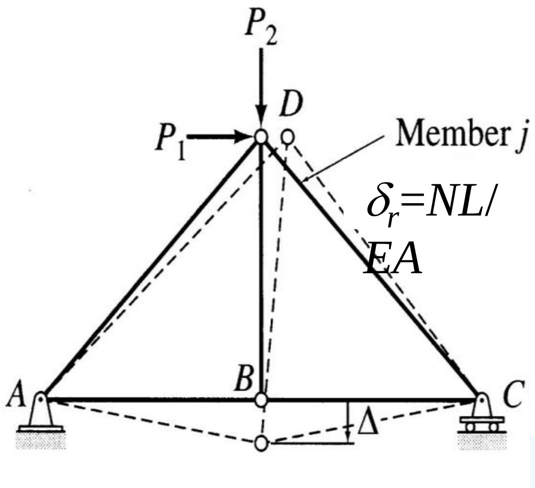
- Deflection of trusses by Work & Strain energy principle
- Principle of Virtual Work
- Deflections of Trusses by the V. W. M.
- Deflections of Beams by the V. W. M.
- Deflections of Frames by the V. W. M.

Deflections of Trusses Under External Loads and Other Effects

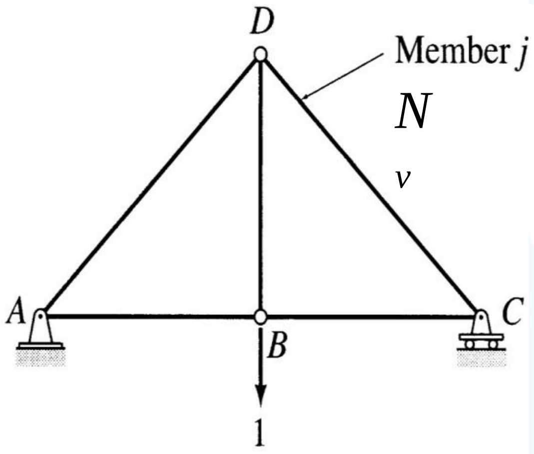
The virtual work method is used to determine the deflections of trusses under the action of external load, *and temperature change or fabrication errors*.

Let us assume that we want to determine the vertical deflection Δ, at joint B of the truss due to the given external Loads P_1 & P_2.

If N represents the internal axial force in an arbitrary member j of the truss then from the axial deformation, δ, of this member is given by: $\delta = NL/EA$, where L, A & E, denote respectively, the length, cross-section and elastic modulus of member j. $W_{ve} = 1_v(\Delta_r)$ $U_{vi} = \Sigma N_v(NL/EA) = \Sigma N_v(\delta_r)$



(a) Real System



(b) Virtual System

1) Under the action of external loads:

$$1_v(\Delta_r) = \Sigma N_v(N_r L/EA)$$

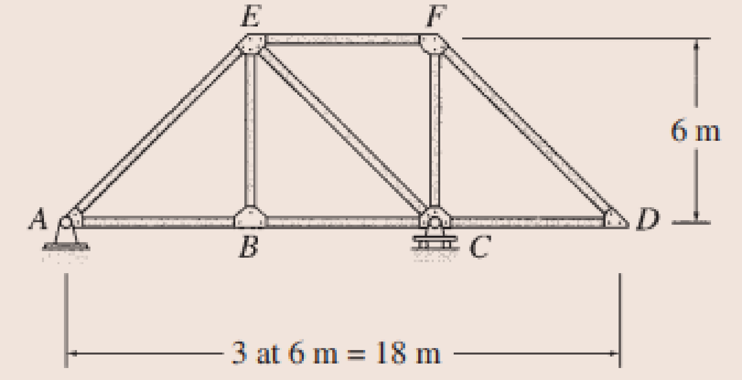
2) Under the action of a temperature change ΔT:

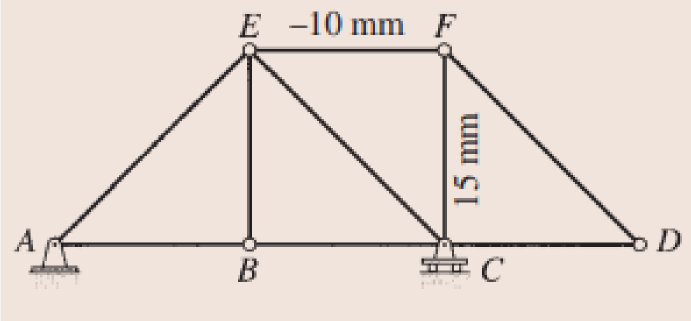
$$1_v(\Delta_r) = \Sigma N_v[\alpha(\Delta T)_r L]$$

3) Under the action of a fabrication error δ_r:

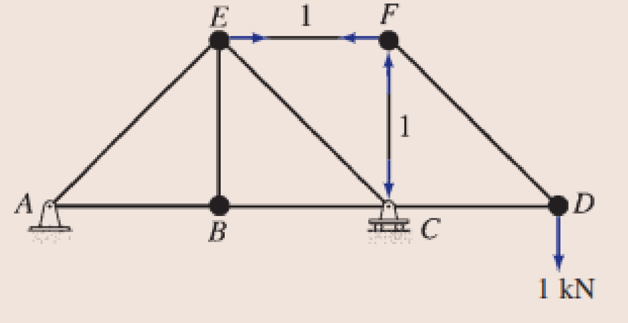
$$1_v(\Delta) = \Sigma N_v(\delta_r)$$

Example 01: Use the virtual work method to determine the vertical deflection at joint D of the truss shown in the following figure if member CF is 15 mm too long and member EF is 10 mm too short.





(b) Real System — δ



(c) Virtual System — F_v Forces

Example 01: The member axial forces due to the real system (N) and this virtual system (N_{v1}) are then tabulated as shown in the following table:.

Member	δ (mm)	N_v (kN)	$N_v\delta$ (kn.mm)
EF	15	-1	-15
CF	-10	1	-10
			-25

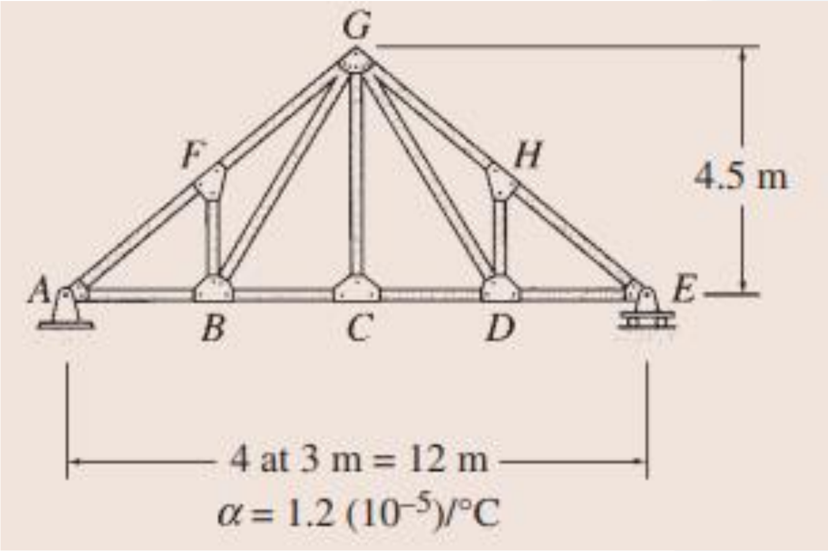
$$1(\Delta_D) = \sum N_v \delta$$

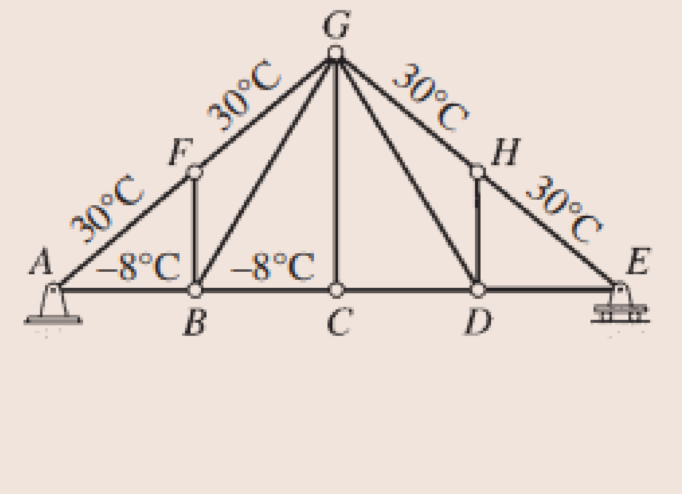
$$(1\ kN)\Delta_D = -25\ kN.mm$$

$$\Delta_D = -25\ mm$$

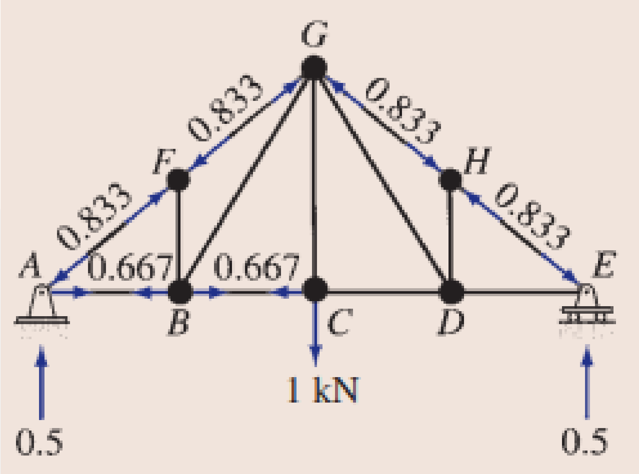
$$\Delta_D = 25\ mm \uparrow$$

Example 02: Use the virtual work method to determine the vertical deflection at joint C of the truss shown in the following due to a temperature drop of 8°C in members AB and BC and a temperature increase of 30°C in members AF, FG, GH, and EH.





(b) Real System — ΔT



(c) Virtual System — F_v Forces

Example 02: The member axial forces due to the real system (N) and this virtual system (N_{v1}) are then tabulated as shown in the following table:.

Member	L (m)	ΔT (°C)	N_v (kN)	$N_{v1}(\Delta T)L$ (kN-°C-m)
AB	3	-8	0.667	-16.0
BC	3	-8	0.667	-16.0
AF	3.75	30	-0.833	-93.7
FG	3.75	30	-0.833	-93.7
GH	3.75	30	-0.833	-93.7
EH	3.75	30	-0.833	-93.7
				-406.8

$$1(\Delta_C) = \propto \sum N_v(\Delta T)L$$

$$(1\ kN)\Delta_C = 1.2\ (10^{-5})(-406.8)$$

$$\Delta_C = -0.00488\ m$$

$$\Delta_C = 4.88\ mm \uparrow$$

Structural Mechanics (1)

Week No-04

Part-02

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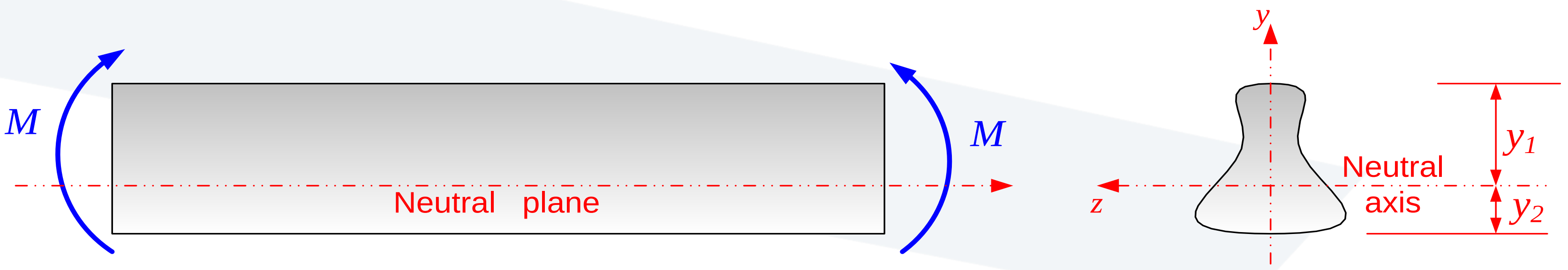
Strain energy in a beam element

Bending Strain energy in a beam element



$$U = \iiint_V \frac{1}{2}\sigma_x \varepsilon_x dV = \iiint_V \frac{\sigma_x^2}{2E} dV = \iiint_V \frac{1}{2E}\left(\frac{M^2}{I^2}y^2\right) dxdA = \int_0^L \frac{1}{2E} dx \left(\frac{M^2}{I^2}\right) \iint_A y^2 dA = \int_0^L \frac{M^2}{2EI} dx$$

Shear Strain energy in a beam element,

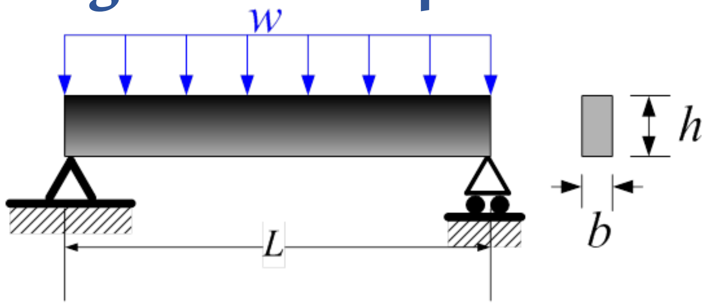
$$U = \iiint_V \frac{1}{2}\tau\gamma dV = \iiint_V \frac{\tau^2}{2G} dV = k\int_0^L \frac{S^2}{2GA} dx$$

$$k = \begin{cases} 1.2 & \text{forarectangle} \\ 1.1 & \text{foracircle} \\ 1.2 & \text{forathin circular} \end{cases}$$

Comparison of bending and shear strain energies in a simple beam

$$U_b = \int_0^L \frac{M^2}{2EI}dx \qquad U_S = 1.2\int_0^L \frac{S^2}{2GA}dx$$

$$M(x) = -\tfrac{1}{2}wx^2 + \tfrac{1}{2}wLx \qquad S(x) = -wx + \tfrac{1}{2}wL$$



$$I = bh^3/12 \quad \& \quad A = bh$$

$$U_b = \int_0^L \frac{M^2}{2EI}dx = \frac{6}{Ebh^3}\int_0^L (-\tfrac{1}{2}wx^2 + \tfrac{1}{2}wLx)^2 dx$$

$$= \frac{6w^2}{Ebh^3}\int_0^L (\tfrac{1}{4}x^4 - \tfrac{1}{2}Lx^3 + \tfrac{1}{4}L^2x^2)dx$$

$$= \frac{6w^2}{Ebh^3}\left[\tfrac{1}{20}x^5 - \tfrac{1}{8}Lx^4 + \tfrac{1}{12}L^2x^3\right]_0^L = \frac{0.05w^2L^5}{Ebh^3}$$

$$U_S = 1.2\int_0^L \frac{S^2}{2GA}dx = \frac{0.6}{Gbh}\int_0^L (wx - \tfrac{1}{2}wL)^2 dx$$

$$= \frac{0.6w^2}{Gbh}\int_0^L (x^2 - Lx + \tfrac{1}{4}L^2)dx$$

$$= \frac{0.6w^2}{Gbh}\left[\tfrac{1}{3}x^3 - \tfrac{1}{2}Lx^2 + \tfrac{1}{4}L^2x\right]_0^L = \frac{0.05w^2L^3}{Gbh}$$

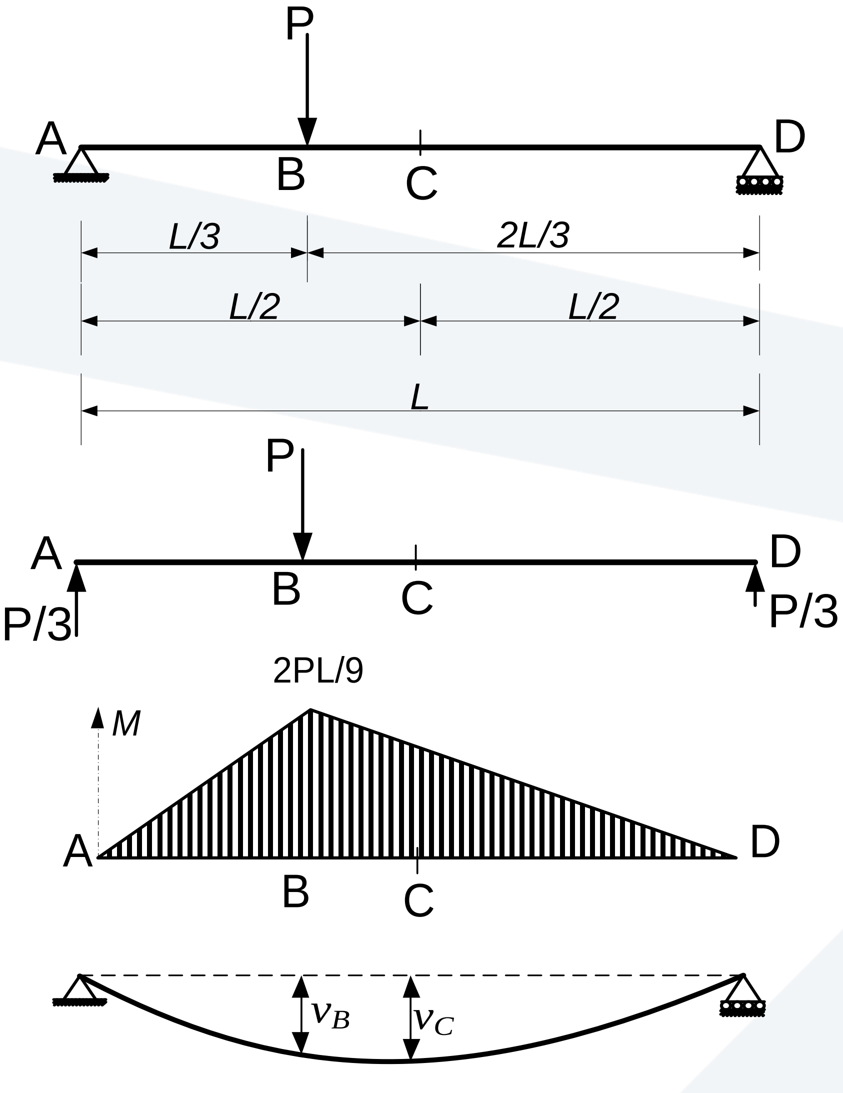
$$U_S / U_b = \left(\frac{0.05w^2L^3}{Gbh}\right) / \left(\frac{0.05w^2L^5}{Ebh^3}\right)$$

$$= (E/G)(h^2/L^2) \approx 2h^2/L^2 \ll 1.$$

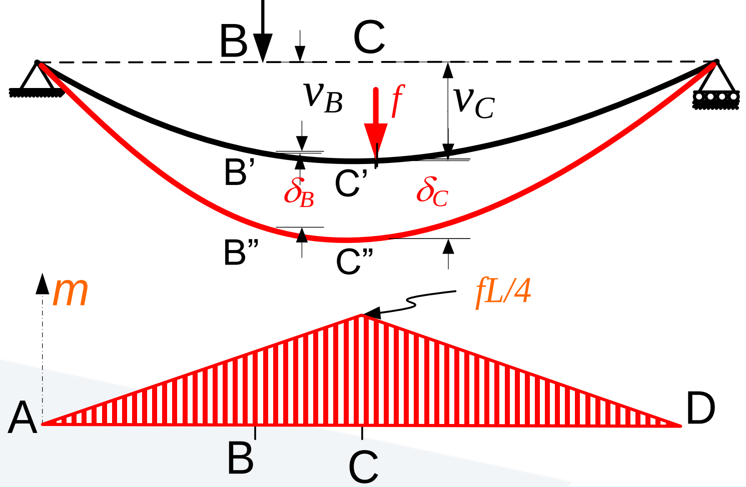
Shear and Axial Strain energies are negligible in comparison with the bending moment energy

B. Haidar

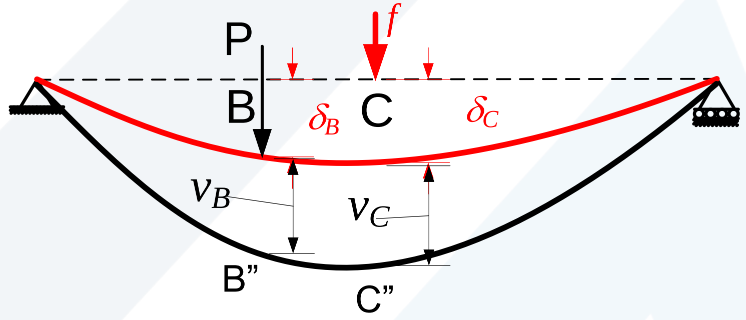
DEFLECTIONS OF BEAMS BY THE V. W. M.



$$\frac{1}{2}Pv_B = \int_0^L \frac{M^2}{2EI}dx$$



$$\frac{1}{2}Pv_B + P\delta_B + \frac{1}{2}f\delta_C = \int_0^L \frac{(M+m)^2}{2EI}dx$$



$$\frac{1}{2}f\delta_C + f\,v_C + \frac{1}{2}Pv_B = \int_0^L \frac{(m+M)^2}{2EI}dx$$

$$P\delta_B = f\,v_C = \int_0^L \frac{mM}{EI}dx$$